Work and Life :
A Hot Entangled Mess
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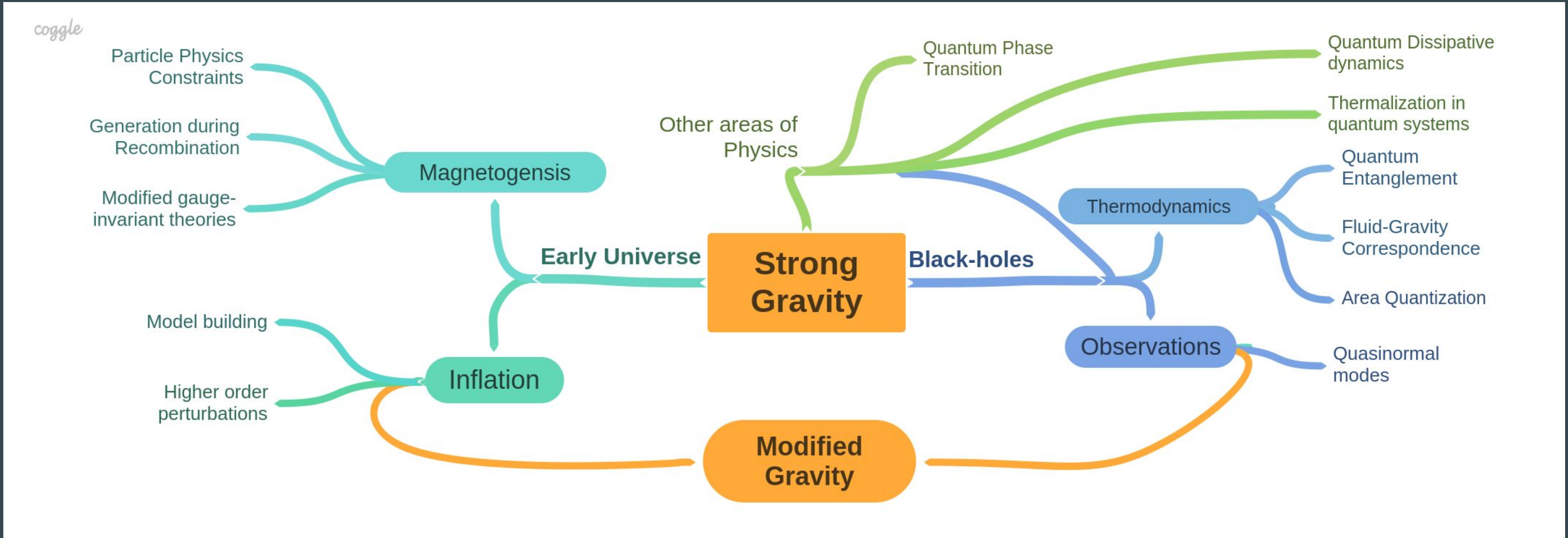
Talk Outline

- Introduction to my work

- Interactive session

Prof. S. Shankaranarayanan's Research



Quantum Entanglement

Quantum entanglement : Basics

- Entanglement : Captures purely quantum correlations between particles regardless of their physical separation.

- Arises from subsystem interaction. For a bi-partite system, Hamiltonian can be broken down as:

$$H = H_1 + H_2 + H_{int}$$

- Examples : Coupled Harmonic Oscillator, Hydrogen Atom

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2 x_1^2 + \frac{1}{2}m\omega^2 x_2^2 + kx_1x_2$$

Quantum entanglement : Basics

- Pure state entanglement : $\Psi(x_1, x_2) \neq \psi_1(x_1)\psi_2(x_2)$

- “Spooky action at a distance”

- Ignorance to information : Von Neumann Entropy / Entanglement entropy

$$S = -Tr\rho_{red} \log \rho_{red}$$

Limits of entanglement entropy

- Full information about the system : $S = 0$
- Otherwise : $S > 0$
- For CHO :

$$k \to 0 \Rightarrow S \to 0 \qquad k \to \infty \Rightarrow S \to \infty$$

- Divergence occurs due to zero modes.

[SMC & SS '19]

Black hole thermodynamics from entanglement

Quantum Entanglement Mechanics of Scalar Field

- Field theory limit : $a \to 0$ & $N \to \infty$

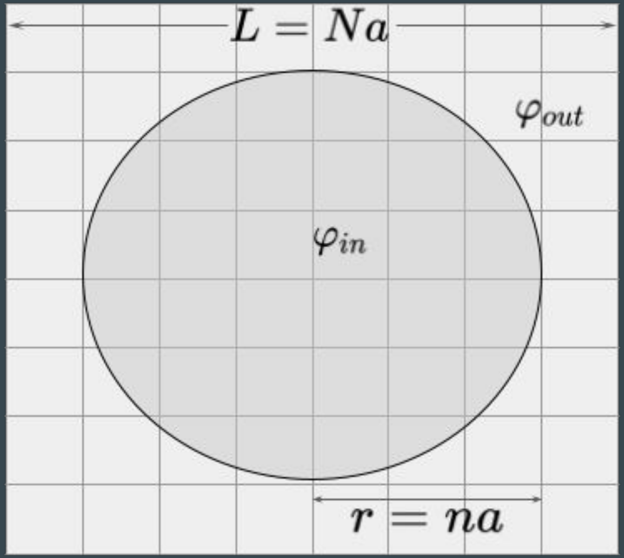
- Entanglement entropy :

$$S = -Tr\,\rho_{red} \log \rho_{red}$$

Follows an area law.

- Entanglement energy :

$$E = \epsilon\, Tr\left[\rho : H_{in} :\right]$$



Black Hole Thermodynamics from Entanglement

- One-to-one correspondence :

$$E = \frac{c_e}{a^2} E_{Komar}; \qquad S = \frac{c_s}{\pi a^2} S_{BH}$$

- Constants c_e and c_s are universal for all space-times, for any horizon.

- Komar relation is satisfied, regardless of UV cut-off:

$$E = 2T_H S \quad \Leftrightarrow \quad E_{Komar} = 2T_H S_{BH}$$

Smarr formula from entanglement

Space-time	Entanglement Structure	Thermodynamic Structure	Smarr formula	Pressure	Potential
Schwarzschild	$S_{\text{ent}} = (c_s/a^2)r_h^2$ $E_{\text{ent}} = (c_e/a^2)M$	$S_{BH} = \pi r_h^2$ $E_{\text{Komar}} = M$	$M = 2T_{\text{H}}\, S_{BH}$	—	—
Reissner-Nordström	$S_+ = (c_s/a^2)r_+^2$ $E_+ = (c_e/a^2)\sqrt{M^2 - Q^2}$	$S_{BH} = \pi r_+^2$ $E_{Komar} = \sqrt{M^2 - Q^2}$	$M = 2T_{\text{H}}\, S_{BH} + Q^2/r_+$	—	Q/r_+
Schwarzschild-AdS	$S_{\text{ent}} = (c_s/a^2)r_h^2$ $E_{\text{ent}} = (c_e/a^2)[3M - r_h^2]$	$S_{BH} = \pi r_h^2$ $E_{Komar} = 3M - r_h^2$	$M = 2T_{\text{H}}\, S_{BH} - r_h^3/l^2$	$3/8\pi l^2$	—
Schwarzschild-dS	$S_b = (c_s/a^2)r_b^2$ $E_b = (c_e/a^2)[3M - r_b^2]$	$S_{BH} = \pi r_b^2$ $E_{Komar} = 3M - r_b^2$	$M = 2T_{\text{H}}\, S_{BH} + r_b^3/l^2$	$-3/8\pi l^2$	—

[SMC & SS '20]

Further applications of quantum entanglement

- Black hole thermalization
- Quantum complexity and chaos
- Signatures of quantum phase transitions
- Black hole information paradox

Thank You

Zero mode divergence

- To decouple the system, shift to the normal mode co-ordinates.
- Suppose there is a free particle in the decoupled system, the entanglement entropy diverges. Also called a zero-mode divergence.
- Reason : Non-normalizability
- Can be extended to hydrogen atom, oscillator networks, etc. Leads to interesting effects such as quantum criticality in field theories.

Hamiltonian of a massless scalar field in flat space-time:

$$H = \frac{1}{2}\int d^3x \left[\pi^2(\mathbf{x}) + |\nabla\varphi(\mathbf{x})|^2\right]$$

$$\Big\downarrow \text{ Regularization Techniques}$$

$$H_{lm} = \frac{1}{2a}\sum_{j=1}^{N}\left[\pi_{lm,j}^2 + \left(j+\frac{1}{2}\right)^2\left(\frac{\varphi_{lm,j}}{j} - \frac{\varphi_{lm,j+1}}{j+1}\right)^2 + \frac{l(l+1)}{j^2}\varphi_{lm,j}^2\right]$$

$$\Big\downarrow$$

$$H_{osc} = \frac{1}{2}\left\{\sum_{i=1}^{N} p_i^2 + \sum_{i,j=1}^{N} x_i K_{ij} x_j\right\}$$

We end up with a system of N harmonic oscillators coupled together.

Static, Spherically Symmetric Space-times

- Line element

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2$$

- Space-time horizon : $f(r_h) = 0$

- Example : Schwarzschild Black Hole $f(r) = 1 - \frac{2M}{r}$

- Another example : Reissner-Nordstrom $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$

Equation of state for black holes : Smarr formula

- For ideal gases : $PV = Nk_BT$

- For black holes :

$$M = 2T_H\, S_{\mathrm{BH}} + 2\Omega_H J + \Phi_H Q$$